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MIKE 21 & MIKE 3 Flow Model FM

Mud Transport Module

Scientific Documentation

DHI headquarters
Agern Allé 5
DK-2970 Hørsholm
Denmark

+45 4516 9200 Telephone
+45 4516 9333 Support
+45 4516 9292 Telefax

mike@dhigroup.com
www.mikepoweredbydhi.com

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1 Introduction

The present Scientific Documentation aims at giving an in-depth description of the theory behind and equations used in the Mud Transport Module of the MIKE 21 & MIKE 3 Flow Model FM.

First a general description of the term "mud" is given. This is followed by a number of sections giving the physical, mathematical and numerical background for each of the terms in the cohesive sediment and fine sand transport equations.

Special sections describe the case of multi-layer and multi-fraction applications as well as the option of morphological simulations.

2 What is Mud

Mud is a term generally used for fine-grained and cohesive sediment with grain-sizes less than 63 microns. Mud is typically found in sheltered areas protected from strong wave and current activity. Examples are the upper and mid reaches of estuaries, lagoons and coastal bays. The sources of the fine-grained sediments may be both fluvial and marine.

Fine-grained suspended sediment plays an important role in the estuarine environment. Fine sediment is brought in suspension and transported by current and wave actions. In estuaries, the transport mechanisms (settling – and scour lag) acting on the fine-grained material tend to concentrate and deposit the fine-grained material in the inner sheltered parts of the area (Van Straaten & Kuenen, 1958; Postma, 1967; and Pejrup, 1988). A zone of high concentration suspension is called a turbidity maximum and will change its position within the estuary depending on the tidal cycle and the input of fresh-water from rivers, etc. (e.g. Dyer, 1986).

Fine sediments are characterised by low settling velocities. Therefore, the sediments may be transported over long distances by the water flow before settling. The cohesive properties of fine sediments allow them to stick together and form larger aggregates or flocs with settling velocities much higher than the individual particles within the floc (Krone, 1986; Burt, 1986). In this way they are able to deposit in areas where the individual fine particles would never settle. The formation and destruction of flocs are depending on the amount of sediment in suspension as well as the turbulence properties of the flow. This is in contrast to non-cohesive sediment, where the particles are transported as single grains.

Figure 2.1 Muddy (left) and sandy (right) sediments

Fine sediment is classified according to grain-size as shown in Table 2.1.

Table 2.1 Classification of fine sediment

Sediment type	Grain size	Flocculation ability
Clay	< 4 μm	high
Silt	4-63 μm	medium
Fine sand	63-125 μm	very low/no flocculation

3 General Model Description

3.1 Introduction

In order to include the transport and deposition processes of fine-grained material in the modelling system, it is necessary to integrate the description with the advection-diffusion equation caused by the water flow.

MIKE 21 & MIKE 3 Flow Model FM is based on a flexible mesh approach and has been developed for applications within oceanographic, coastal and estuarine environments. In the case of 2D, the model is depth-integrated. This means that the simulation of the transport of fine-grained material must be averaged over depth and appropriate parameterisations of the sediment processes must be applied.

In the MIKE 21 & MIKE 3 Flow Model FM model complex, the transport of fine-grained material (mud) has been included in the Mud Transport module (MT), linked to the Hydrodynamic module (HD), as indicated in Figure 3.1.

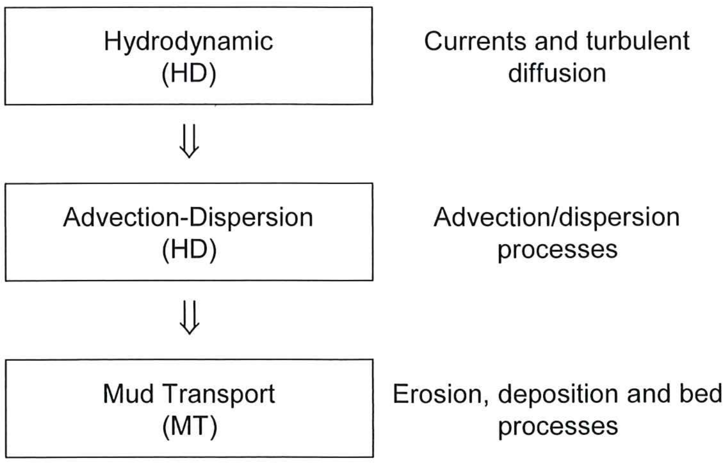

Figure 3.1 Data flow and physical processes for MIKE 21 & MIKE 3 Flow Model FM, Mud Transport calculation

The processes included in the Mud Transport module are kept as general as possible. The Mud Transport module includes the following processes:

- Multiple mud fractions
- Multiple bed layers
- Wave-current interaction
- Flocculation
- Hindered settling
- Inclusion of a sand fraction
- Transition of sediments between layers
- Simple morphological calculations

The above possibilities cover most cases appropriate for 2D modelling. In case special applications are required such as simulating the influence of high sediment concentrations on the water flow through formation of stratification and damping of turbulence, the modeller is referred to 3D modelling.

3.2 Governing Equations

The sediment transport formulations are based on the advection-dispersion calculations in the Hydrodynamic module.

The Mud Transport module solves the so-called advection-dispersion equation:

$$\frac{\partial \bar{c}}{\partial t} + u\frac{\partial \bar{c}}{\partial x} + v\frac{\partial \bar{c}}{\partial y} =$$

$$\frac{1}{h}\frac{\partial}{\partial x}\left(hD_x\frac{\partial \bar{c}}{\partial x}\right) + \frac{1}{h}\frac{\partial}{\partial y}\left(hD_y\frac{\partial \bar{c}}{\partial y}\right) + Q_L C_L \frac{1}{h} - S \tag{3.1}$$

where

$\bar{c}$	depth averaged concentration (g/m^3)
u,v	depth averaged flow velocities (m/s)
D_x,D_y	dispersion coefficients (m^2/s)
h	water depth (m)
S	deposition/erosion term (g/m^3/s)
Q_L	source discharge per unit horizontal area (m^3/s/m^2)
C_L	concentration of the source discharge (g/m^3)

In cases of multiple sediment fractions, the equation is extended to include several fractions while the deposition and erosion processes are connected to the number of fractions.

3.3 Numerical Schemes

The advection-dispersion equation is solved using an explicit, third-order finite difference scheme, known as the ULTIMATE scheme (Leonard, 1991). This scheme is based on the well-known QUICKEST scheme (Leonard, 1979; Ekebjærg & Justesen, 1991).

This scheme has been described in various papers dealing with turbulence modelling, environmental modelling and other problems involving the advection-dispersion equation. It has several advantages over other schemes, especially that it avoids the "wiggle" instability problem associated with central differentiation of the advection terms. At the same time it greatly reduces the numerical damping, which is characteristic of first-order up-winding methods.

The scheme itself is a Lax-Wendroff or Leith-like scheme in the sense that it cancels out the truncation error terms due to time differentiation up to a certain order by using the basic equation itself. In the case of QUICKEST, truncation error terms up to third-order are cancelled for both space and time derivatives.

The solution of the erosion and the deposition equations are straightforward and do not require special numerical methods.

4 Cohesive Sediments

4.1 Introduction

The Mud Transport module of MIKE 21 & MIKE 3 Flow Model FM describes the erosion, transport and deposition of fine-grained material < 63 μm (silt and clay) under the action of currents and waves. For a correct solution of the erosion processes, the consolidation of sediment deposited on the bed is also included. The model is essentially based on the principles in Mehta et al. (1989) with the innovation of including the bed shear stresses due to waves.

Clay particles have a plate-like structure and an overall negative ionic charge due to broken mineral bonds on their faces. In saline water, the negative charges on the particles attract positively charged cations and a diffuse cloud of cations is formed around the particles. In this way the particles tend to repel each other (Van Olphen, 1963). Still, particles in saline water flocculate and form large aggregates or flocs in spite of the repulsive forces. This is because in saline water, the electrical double layer is compressed and the attractive van der Waals force acting upon the atom pairs in the particles becomes active. Flocculation is governed by increasing concentration, because more particles in the water enhance meetings between individual particles. Turbulence also plays an important role for flocculation both for the forming and breaking up of flocs depending on the turbulent shear (Dyer, 1986).

A deterministic physically based description of the behaviour of cohesive sediment has not yet been developed, because the numerous forces included in their behaviour tend to complicate matters. Consequently, the mathematical descriptions of erosion and deposition are essentially empirical, although they are based on sound physical principles.

The lack of a universally applicable, physically based formulation for cohesive sediment behaviour means that any model of this phenomenon is heavily dependent on field data (Andersen & Pejrup, 2001; Andersen, 2001; Edelvang & Austen, 1997; Pejrup et al., 1997). Extensive data over the entire area to be modelled is required such as:

- bed sedimentology
- bed erodibility
- biology
- settling velocities
- suspended sediment concentrations
- current velocities
- vertical velocity and suspended sediment concentration profiles
- compaction of bed layers
- effect of wave action
- critical shear stresses for deposition and erosion

Of course, the dynamic variation of water depth and flow velocities must also be known along with boundary values of suspended sediment concentration.

The Mud Transport module consists of a 'water-column' and an 'in-the-bed' module. The link between these two modules is source/sink terms in an advection-dispersion model.

The transport and deposition of fine-grained material is governed by the fact that settling velocities are generally slow compared to sand. Hence, the concentration of suspended material does not adjust immediately to changes in the hydraulic conditions. In other words, the sediment concentration at a given time and location is dependent on the conditions upstream of this location at an earlier time. Postma (1967) first described this process, called settling- and scour-lag. This is the main factor for the concentration of fine material in estuaries often resulting in a turbidity maximum. In order to describe this process, the sediment computation has been built into the advection-dispersion formulation in the Hydrodynamic module.

For 3D calculations the viscosity and density may be influenced by large concentrations of mud in the water column.

The source and sink term S in the advection-dispersion equation depends on whether the local hydrodynamic conditions cause the bed to become eroded or allow deposition to occur. Empirical relations are used, and possible formulations for evaluating S are given below.

The mobile suspended sediment is transported by long-period waves only, which are tidal currents, whereas the wind-waves are considered as "shakers". Combined they are able to re-entrain or re-suspend the deposited or consolidated sediment.

The processes in the bed are described in a multi-layer bed; each layer is described by a critical shear stress for erosion, erosion coefficient, power of erosion, density of dry sediment and erosion function.

The bed layers can be soft and partly consolidated or dense and consolidated.

Consolidation is included as a transition rate of sediment between the layers and liquefaction by waves is included as a weakening of the bed due to breakdown of the bed structure.

A conceptual illustration of the physical processes modelled by a "multi bed layer approach" is shown in Figure 4.1.

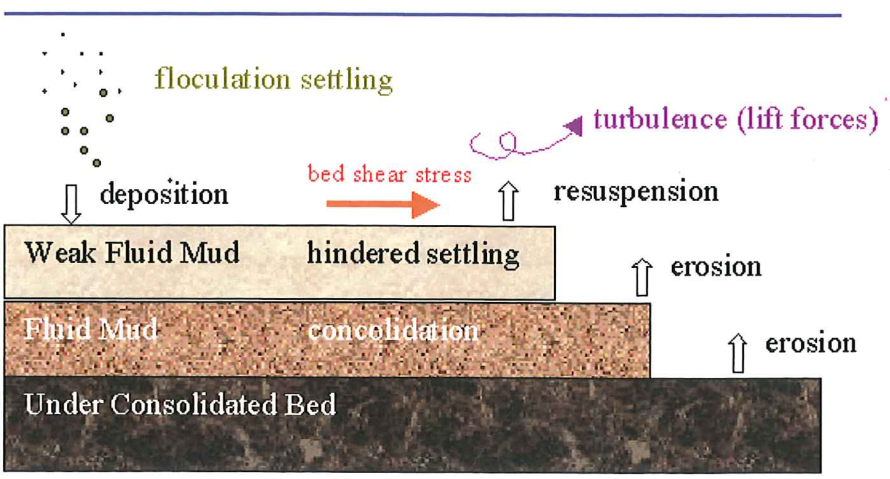

Figure 4.1 Multi-layer model and physical processes for example with three bed layers

4.2 Deposition

In the MT model, a stochastic model for flow and sediment interaction is applied. This approach was first developed by Krone (1962).

Krone suggests that the deposition rate can be expressed by:

$$Deposition\ :\ S_D = w_s c_b p_d \tag{4.1}$$

where

w_s	settling velocity (m/s)
c_b	near bed concentration (kg/m^3)
p_d	probability of deposition

The probability of deposition p_d is calculated as:

$$p_d = 1 - \frac{\tau_b}{\tau_{cd}} = \tau_b \le \tau_{cd} \tag{4.2}$$

where

τ_b	the bed shear stress (N/m^2)
τ_{cd}	the critical bed shear stress for deposition (N/m^2)

4.3 Settling Velocity and Flocculation

The settling velocity of the fine sediment depends on the particle/floc size, temperature, concentration of suspended matter and content of organic material.

Usually one distinguishes between a regime where the settling velocity increases with increasing concentration (flocculation) and a regime where the settling velocity decreases with increasing concentration. The latter is referred too as hindered settling. The first is the most common of the two in the estuary.

Following Burt (1986), the settling velocity in saline water (>5 ppt) can be expressed by:

$$w_s = kc^{\gamma}\ \ for\ c \le 10 kg/m^3 \tag{4.3}$$

where

w_s	settling velocity of flocs (m/s)
c	volume concentration
k,γ	coefficients
γ	1 to 2

The relation for $c \le 10$ kg/m^3 describes the flocculation of particles based on particle collisions. The higher concentration the higher possibility for the particles to flocculate.

$c > 10$ kg/m^3 corresponds to 'hindered' settling, where particles are in contact with each other and do not fall freely through the water.

Alternative settling formulations are also available:

The formulation of Richardson and Zaki (1954) is the classical equation for hindered settling:

$$w_s = w_{s,r}\left(1 - \frac{c}{c_{gel}}\right)^{w_{s,n}} \tag{4.4}$$

Where $w_{s,r}$ is a reference value, $w_{s,n}$ a coefficient and c_{gel} the concentration at which the flocs start to form a real self-supported matrix (referred to as the gel point).

Winterwerp (1999) proposed the following for hindered settling:

$$w_s = w_{s,r}\frac{(1-\Phi_*)(1-\Phi_p)}{1+2.5\Phi} \tag{4.5}$$

where

$$\Phi_p = \frac{c}{\rho_s} \tag{4.6}$$

and ρ_s is the density of sediment grains.

Flocculation is enhanced by high organic matter content including organic coatings, etc. (Van Leussen, 1988; Eisma, 1993). In fresh water, flocculation is dependent on organic matter content, whereas in saline waters salt flocculation also occurs. The influence of salt on flocculation is primarily important in areas where fresh water meets salt water such as estuaries. The following expression is used to express the variation of settling velocity with salinity. Please note that the reference value w_s is the value representative for saline water:

$$w_s = w_s\left(1 - C_1 e^{C_2}\right) \tag{4.7}$$

where C_1 and C_2 are calibration parameters.

Figure 4.2 shows an example of C_1 ={0 , 0.5 , 1} and C_2 = -1/3.

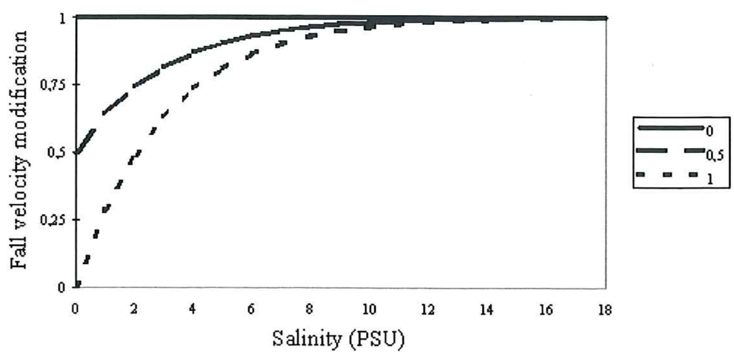

Figure 4.2 Settling velocity and salinity dependency

The description of salt flocculation is based on Krone's experimental research (Krone, 1962) Whitehouse et al., (1960) studied the effect of varying salinities on flocculation of different clay minerals in the laboratory. Gibbs (1985) showed that in the natural environment, flocculation is more dependent on organic coating. Therefore, the effect of mineral constitution of the sediment is not taken into account in the model.

4.4 Sediment Concentration Profiles

Two expressions for the sediment concentration profile can be applied. Either an expression that is based on an approximate solution to the vertical sediment fluxes during deposition (Teeter) or an expression that assumes equilibrium between upward and downward sediment fluxes (Rouse).

The difference between the two expressions is depicted in Figure 4.3.

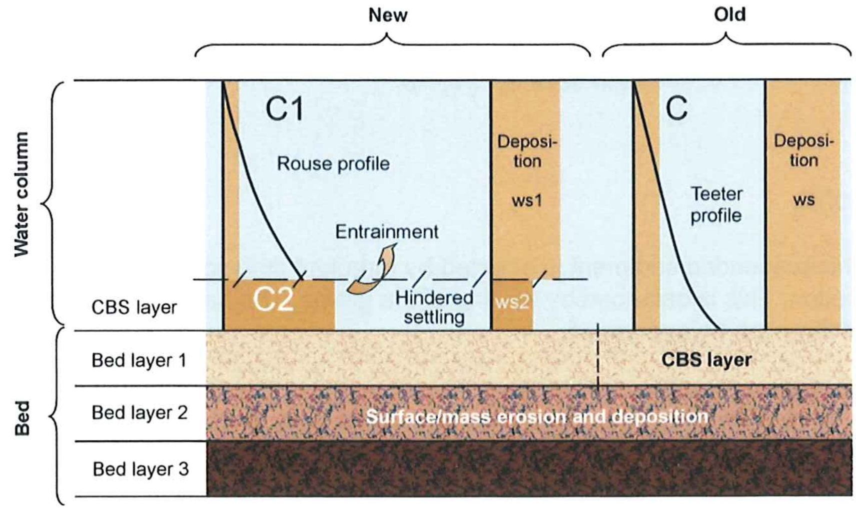

Figure 4.3 Definition of Rouse profile vs. Teeter profile

The near bed concentration c_b is proportional to the depth averaged mass concentration $\bar{c}$ and is related to the vertical transport, i.e. a ratio of the vertical convective and diffusive transport represented by the Peclet number P_e:

$$P_e = \frac{C_{rc}}{C_{rd}} \tag{4.8}$$

where

C_{rc}	convective Courant number = $w_s \; \Delta t / h$
C_{rd}	diffusive Courant number = $\bar{D}_z \; \Delta t / h^2$
w_s	mean settling velocity of the sediment
$\bar{D}_z$	depth mean eddy diffusivity

4.4.1 Teeter 1986 profile

In this expression the near bed concentration c_b is related to the depth averaged concentration $\bar{c}$ (Teeter, 1986):

$$\beta = \frac{c_b}{\bar{c}} \tag{4.9}$$

where

$$\beta = 1 + \frac{P_e}{1.25 + 4.75 p_d^{2.5}} \tag{4.10}$$

$$P_e = \frac{w_s h}{\overline{D_z}} = \frac{6 w_s}{\kappa U_f} \tag{4.11}$$

κ	Von Karman's universal constant (κ = 0.4)
U_f	Friction velocity $\sqrt{\tau_b / \rho}$

4.4.2 Rouse profile

The suspended sediment is affected by turbulent diffusion, which results in an upward motion. This is balanced by settling of the grains. The balance between diffusion and settling can be expressed:

$$-\varepsilon \frac{dC}{dz} = w_s C \tag{4.12}$$

where

ε	diffusion coefficient
C	concentration as function of z
z	vertical Cartesian coordinate

By assuming that ε is equal to the turbulent eddy viscosity, and applying the parabolic eddy viscosity distribution

$$\varepsilon = \kappa U_f z \left(1 - \frac{z}{h}\right) \tag{4.13}$$

where

h	height of water column

the following vertical concentration profile will be given:

$$C = C_a \left[\frac{a}{h-a} \frac{h-z}{z} \right]^R, a \le z \le h \tag{4.14}$$

where

C_a	reference concentration at $z = a$
a	reference level
R	Rouse number

$$R = \frac{w_s}{\kappa U_f} \tag{4.15}$$

It is possible to choose a vertical variation of the concentration of suspended sediment in order to determine the settling distance. The average depth, h^* through which the particles settle at deposition is given by:

$$\frac{h_*}{h} = \frac{\int_0^1 s \left(\frac{1}{s} - 1 \right)^R ds}{\int_0^1 \left(\frac{1}{s} - 1 \right)^R ds} \tag{4.16}$$

where $s = h/z$ and the term $\frac{h_*}{h}$ is named the relative height of centroid.

The near bed suspended sediment concentration c_b is related to the depth-averaged concentration $\bar{c}$ using the Rouse profile

$$c_b = \frac{\bar{c}}{RC} \tag{4.17}$$

in which RC is the relative height of centroid.

4.5 Erosion

Erosion can be described in two ways depending upon whether the bed is dense and consolidated (Partheniades, 1965) or soft and partly consolidated (Parchure and Mehta, 1985).

For dense, consolidated bed the erosion (S_E) is defined by:

$$S_E = E \left(\frac{\tau_b}{\tau_{ce}} - 1 \right)^n, \tau_b > \tau_{ce} \tag{4.18}$$

where

E	erodibility of bed (kg/m^2/s)

τ_b bed shear stress (N/m^2)

τ_{ce} critical bed shear stress for erosion (N/m^2)

n power of erosion

For soft, partly consolidated bed the erosion is defined by:

$$S_E = E \exp\left[\alpha(\tau_b - \tau_{ce})^{1/2}\right], \tau_b > \tau_{ce} \tag{4.19}$$

where

α coefficient ($m/\sqrt{N}$)

4.6 Bed Description

It is possible to describe the bed as having more than one layer. Each layer is described by the critical shear stress for erosion, $\tau_{ce,j}$, power of erosion, n_j, density of dry bed material, ρ_i, erosion coefficient, E_j, and α_j-coefficient. The deposited sediment is first included in the top layer. The layers represent weak fluid mud, fluid mud and under-consolidated bed (Mehta et al., 1989) and are associated with different time scales.

The model requires an initial thickness of each layer to be defined.

The consolidation process is described as the transition of sediment between the layers (Teisson, 1992; Sanford and Maa, 2001).

The influence of waves is taken into account as liquefaction resulting in a weakening of the bed due to breakdown of bed structure. This may cause increased surface erosion, because of the reduced strength of the bed top layer (Delo and Ockenden, 1992).

5 Fine Sand Sediment Transport

5.1 Introduction

The major difference between the description of cohesive sediment transport and sand transport is the distinction of the suspended sediment concentration profile. In the Mud Transport module, a simple description of the vertical concentration profile is applied. The time-scale needed to deform the profile by the flow-conditions is long in comparison to a concentration profile of sand, which is primarily transported as bed-load.

However, it is possible to activate sand transport formulations in the Mud Transport module in case a certain percentage of the bed material is in the fine sand fraction between 63 and 125 μm that may be transported both in suspension and as bed load. These formulations are built into the advection-dispersion formulation.

5.2 Suspended Load Transport

The equilibrium concentration $\bar{c}_e$ is defined as:

$$\bar{c}_e = \frac{q_s}{\bar{u}h} \tag{5.1}$$

where

$\bar{u}$	is the depth averaged flow velocity
q_s	suspended load transport (kg/m/s)

The suspended load transport is found as the integral of the current velocity profile, *u*, and the concentration profile of suspended sediment, *c*:

$$q_s = \int_a^h c \cdot dy \tag{5.2}$$

where

c	concentration of sediment (kg/m^3) at distance *y* from bed
u	flow velocity (m/s) at distance y from bed
h	water depth (m)
a	thickness of the bed layer (m)

Usually little is known about the bed layer, such as the height of the bed forms. This results in the approximation:

$$a = k_s = 2d_{50} \tag{5.3}$$

where

k_s	equivalent roughness height (m)

d_{50} 50 percent fractile of grain-size of sediment (m)

In the advection-dispersion model, the suspended transport is calculated based on depth-averaged flow velocities, $\bar{u}, \bar{v}$, a compound concentration, $\bar{c}$, and the water depth, *h*, (approx. $h \approx h - a$). The sand transport is described through a depth-averaged equilibrium concentration, $\bar{c}_e$ and an accretion (deposition), erosion term, *S*, which means that no bed transport takes place.

The transport is highly dependent upon two parameters, namely the settling velocity, w_s, and the turbulent sediment diffusion coefficient, ε_s, because these parameters have an effect on both the flow velocity and concentration profile. For a normal sediment load the effect on the velocity profile is negligible.

5.2.1 Requirements for sediments in suspension

The following description is mainly based on Van Rijn (1982, 1984), Yalin (1972) and Engelund and Fredsøe (1976).

The presence of sediment suspension demands that the actual friction velocity, U_f, is larger than a so-called critical friction velocity, $U_{f,cr}$, and that the vertical turbulence is sufficient to create vertical velocity components higher than the settling velocity.

The first assumption is expressed through the transport stage parameter, *T:*

$$T = \begin{cases} \left(\dfrac{U_f}{U_{f,cr}}\right)^2 - 1, & U_f > U_{f,cr} \\ 0, & U_f \le U_{f,cr} \end{cases} \tag{5.4}$$

where $U_{f,cr}$ is found from Shields curve, see Rijn (1982), using the input parameters, d_{50}, relative density of sediment, *s*, and the dimensionless grain size, d^*, defined as the cube root of the ratio of immersed weight to viscous forces:

$$d^* = d_{50}\left[\frac{(s-1)g}{v^2}\right]^{1/3} \tag{5.5}$$

where *n* is the kinematic viscosity of water (m^2/s).

The friction velocity, U_f, reads:

$$U_f = \sqrt{ghI} = \frac{\sqrt{g}}{C_z}\left|\vec{V}\right| \tag{5.6}$$

where

I energy gradient (slope)

C_z Chezy Number ($m^{1/2}/s$) (= 18 ln ($4h/d_{90}$))

d_{90}	90 percent fractile of grain size of sediment (m)
$\left|\vec{V}\right|$	flow speed (m/s)

The second assumption is expressed through some relations between the critical friction velocity, $U_{f,crs}$ for initiation of suspension, the settling velocity, w_s and d^*:

$$\frac{U_{f,crs}}{w_s} = \begin{cases} \dfrac{4}{d^*}, & 1 < d^* \le 10 \\ 0.4 & d^* > 10 \end{cases} \tag{5.7}$$

5.3 Settling Velocity

The settling velocity for fine sand depends on the particle size, viscosity and sediment density.

The settling velocity is expressed by:

$$w_s = \begin{cases} \dfrac{(s-1)gd^2}{18v} & , \quad d < 100\mu m \\ \dfrac{10v}{d}\left\{\left[1+\dfrac{0.01(s-1)gd^3}{v^2}\right]^{0.5} - 1\right\} & , \quad 100 < d \le 1000\mu m \\ 1.1\left[(s-1)gd\right]^{0.5} & , \quad d > 1000\mu m \end{cases} \tag{5.8}$$

where

d	grain size
s	sediment density
v	viscosity
g	acceleration due to gravity

5.4 Sediment Concentration Profiles

The concentration profile is dependent upon the turbulent sediment diffusion coefficient, ε_s, and the settling velocity, w_s. When calculating mud transport $\varepsilon_s = \varepsilon_f$ is assumed, where ε_f is the turbulent flow diffusion coefficient, whereas for fine sand transport the assumption is:

$$\varepsilon_s = \beta \, \Phi \varepsilon_f \tag{5.9}$$

where

β	factor, which describes the difference in the diffusion of a discrete sediment particle and the diffusion of a fluid "particle".
Φ	factor, which expresses the damping of the fluid turbulence by the sediment particles. Dependent upon local sediment concentration.

5.4.1 Diffusion factor

The interpretation of β is not quite clear. Some think that $\beta < 1$, because particles cannot respond fully to turbulent velocity fluctuations. However, some think that $\beta > 1$, because in the turbulent flow the centrifugal forces on the sediment particles would be greater than those on the fluid particles, thereby causing the sediment particles to be thrown to the outside of the eddies with a consequent increase in the effective mixing length and diffusion rate. In this model the following is used:

$$\beta = \begin{cases} 1 + \left(\dfrac{w_s}{U_f}\right)^2 & , \quad \dfrac{w_s}{U_f} < 0.5 \\ 1 & , \quad 0.5 \le \dfrac{w_s}{U_f} < 2.5 \\ \text{no suspension} & , \quad \dfrac{w_s}{U_f} \ge 2.5 \end{cases} \tag{5.10}$$

5.4.2 Damping factor

The Φ factor expresses the influence of the sediment particles on turbulence structure (damping effects) of the fluids.

In order to describe the concentration profile the following equation shall be solved:

$$\frac{dc}{dz} = \frac{w_s c (1-c)^5}{\varepsilon_s} \tag{5.11}$$

The determination of Φ and the solution of this equation are very time-consuming, which leads to a more simplified method, which is chosen in this model.

5.4.3 Concentration profiles

The distribution of the concentration profile is described by the Peclet number, P_e:

$$P_e = \frac{C_{rc}}{C_{rd}} \tag{5.12}$$

where

C_{rc}	convective Courant number (= $w_s\, \Delta t/h$)
C_{rd}	diffusive Courant number (= $\varepsilon_f\, \Delta t/h^2$)
ε_f	depth averaged fluid diffusion coefficient

This Peclet number is also called a suspension parameter, Z:

$$Z = \frac{w_s}{\beta \kappa U_f} \tag{5.13}$$

where

Z	suspension parameter
κ	Von Karman's universal constant (κ = 0.4)
β	factor (as described in Eq. (5.10) above)

To take into account effects other than those caused by the β factor, a modified suspension parameter, Z' is defined as:

$$Z' = Z + \varphi \tag{5.14}$$

where φ is an overall correction factor, which reads:

$$\varphi = 2.5 \left(\frac{w_s}{U_f} \right)^{0.8} \left(\frac{c_a}{c_0} \right)^{0.4} \tag{5.15}$$

where

c_a	concentration at reference level, $z = a$
c_0	concentration at bed, $z = 0$

The c_a/c_0 concentration ratio is found through the following profiles:

$$\frac{C}{C_a} = \begin{cases} \dfrac{c}{c_a} = \left[\dfrac{a(h-z)}{z(h-a)} \right]^Z & , \quad \dfrac{z}{h} < 0.5 \\ \dfrac{c}{c_a} = \left[\dfrac{a}{h-a} \right]^Z \exp\left(-4Z \left(\dfrac{z}{h} - 0.5 \right) \right) & , \quad \dfrac{z}{h} \geq 0.5 \end{cases} \tag{5.16}$$

c_a is based on measured and computed concentration profiles, and reads:

$$c_a = 0.015 \frac{d_{50}}{a} \frac{T^{1.5}}{d_*^{0.3}} \tag{5.17}$$

The equilibrium concentration, $\bar{c}_e$ in the advection-dispersion equations reads:

$$\bar{c}_e = 10^6 \cdot F \cdot C_a \cdot s \tag{5.18}$$

where *F* is a relation between the bottom concentration and the mean concentration based on numerical integration of the suspended concentration profile expressed by the ratio c/c_a previously mentioned, and *s* the relative density equal to 2.65.

If you use a scale factor, $\bar{c}_e$ is multiplied with this factor.

5.5 Deposition

Deposition is described by:

$$S_d = -\left(\frac{\bar{c}_e - \bar{c}}{t_s}\right), \quad \bar{c}_e < \bar{c} \tag{5.19}$$

where t_s is a time-scale given by

$$t_s = \frac{h_s}{w_s} \tag{5.20}$$

h_s is equal to h_* described in the previous section.

5.6 Erosion

Erosion is described by:

$$S_e = -\left(\frac{\bar{c}_e - \bar{c}}{t_s}\right), \quad \bar{c}_e > \bar{c} \tag{5.21}$$

6 Hydrodynamic Variables

6.1 Bed Shear Stress

The sediment transport formulas described above apply hydrodynamic variables for describing the bed shear stress. This must be determined for pure current or a combined wave-current motion.

6.1.1 Pure currents

In the case of a pure current motion the flow resistance is caused by the roughness of the bed. The bed shear stress under a current is calculated using the standard logarithmic resistance law:

$$\tau_c = \tfrac{1}{2}\rho f_c V^2 \tag{6.1}$$

where

τ_c	bed shear stress (N/m2)
ρ	density of fluid (kg/m3)
V	mean current velocity (m/s)
f_c	current friction factor

$$f_c = 2\left(2.5\left(In\left(\frac{30h}{k}\right)-1\right)\right)^{-2} \tag{6.2}$$

h	water depth (m)
k	bed roughness (m)

6.1.2 Pure wave motion

In the case of pure wave motion, the mean bed shear stress reads:

$$\tau_w = \tfrac{1}{2}\rho f_w U_b^2 \tag{6.3}$$

where

τ_w	bed shear stress
f_w	wave friction factor
U_b	horizontal mean wave orbital velocity at the bed (m/s)

$$U_b = \frac{2H_s}{T_z}\frac{1}{\sinh\left(\frac{2\pi}{L}h\right)} \tag{6.4}$$

H_s	significant wave height (m)
T_z	zero-crossing wave period (s)

An explicit approximation given by Swart (1974) for the wave friction factor is used:

$$f_w = 0.47$$
$$f_w = \exp\left(5.213\left(\frac{a}{k}\right)^{-0.194} - 5.977\right), 1 < \frac{a}{k} \le 3000 \tag{6.5}$$

where

a	horizontal mean wave orbital motion at bed (m)

$$a = \frac{H_s}{\pi}\frac{1}{\sinh\left(\frac{2\pi}{L}h\right)} \tag{6.6}$$

An explicit expression of the wave length is given by Fenton and McKee (1990):

$$L = \frac{gT_z^2}{2\pi}\left(\tanh\left[\frac{2\pi}{T_z}\sqrt{\frac{h}{g}}\right]^{3/2}\right)^{2/3} \tag{6.7}$$

6.1.3 Combined currents and waves

Three wave-current shear stress formulations are offered.

Soulsby et al

Two of the formulations use a parameterised version of Fredsøe (1984) derived by Soulsby et al. 1993.

The default option for the parameterised model is to calculate and use the mean shear stress. Another option is to calculate and use the maximum shear stress. The mean shear stress and maximum shear stress are given below (Soulsby et al., 1993):

$$\frac{\tau_{mean}}{\tau_c + \tau_w} = \frac{\tau_c}{\tau_c + \tau_w}\left(1 + b\left(\frac{\tau_c}{\tau_c + \tau_w}\right)^p\left(1 - \frac{\tau_c}{\tau_c + \tau_w}\right)^q\right)$$
$$\frac{\tau_{max}}{\tau_c + \tau_w} = 1 + a\cdot\left(\frac{\tau_c}{\tau_c + \tau_w}\right)^m\left(1 - \frac{\tau_c}{\tau_c + \tau_w}\right)^n \tag{6.8}$$

where

τ_c	current alone shear stress
τ_w	wave alone shear stress amplitude
b, p, q, a, m, n	constants, which vary for different wave-current theories parameterised

For the Fredsøe (1984) model, these constants are:

$$b = b_1 + b_2 |\cos\gamma|^j + \left(b_3 + b_4 |\cos\gamma|^j\right)\log_{10}(r)$$

$$p = p_1 + p_2 |\cos\gamma|^j + \left(p_3 + p_4 |\cos\gamma|^j\right)\log_{10}(r)$$

$$q = q_1 + q_2 |\cos\gamma|^j + \left(q_3 + q_4 |\cos\gamma|^j\right)\log_{10}(r)$$

$$a = a_1 + a_2 |\cos\gamma|^i + \left(a_3 + a_4 |\cos\gamma|^i\right)\log_{10}(r)$$

$$m = m_1 + m_2 |\cos\gamma|^i + \left(m_3 + m_4 |\cos\gamma|^i\right)\log_{10}(r)$$

$$n = n_1 + n_2 |\cos\gamma|^i + \left(n_3 + n_4 |\cos\gamma|^i\right)\log_{10}(r)$$

where a_1, a_2, etc. are given in the table below, γ is the angle between waves and currents, $i = 0.8$, $j = 3.0$ and $r = 2\, f_w / f_c$.

Table 6.1 Constants for wave-current shear stress formulations

	a	m	n	b	p	q
1	-0.06	0.67	0.75	0.29	-0.77	0.91
2	1.70	-0.29	-0.27	0.55	0.10	0.25
3	-0.29	0.09	0.11	-0.10	0.27	0.50
4	0.29	0.42	-0.02	-0.14	0.14	0.45

Fredsøe

A third option is to calculate and use the bed shear stress found from Fredsøe (1981).

For combined wave-current motion the eddy viscosity is strongly increased in the wave boundary layer close to the bed, and the near bed current profile is retarded.

The effect on the outer current velocity profile is described by introducing a "wave" roughness, k_w, which is larger than the actual bed roughness.

It is assumed that the wave motion is dominant close to the bed compared to the current, which means that the wave boundary layer thickness, δ_w, and wave friction, f_w, can be determined by considering the wave parameters only. The wave boundary thickness, δ_w, is found by (Johnsson and Carlsen, 1976):

$$\delta_w = 0.072k\left(\frac{a}{k}\right)^{0.75} \qquad (6.9)$$

The velocity profile outside the wave boundary layer, which is influenced by the wave boundary layer is given by:

$$\frac{U(z)}{U_{fc}} = 2.5 \cdot \ln\left(\frac{30z}{k_w}\right) \quad (6.10)$$

Where

$U(z)$	Velocity at vertical coordinate z (m/s)
U_{fc}	Friction velocity (m/s)
z	vertical coordinate (m)
k_w	wave roughness

In case of weak wave motion, where the wave roughness k_w is less than the bed roughness k for pure current motion, the latter will be used.

The max bed shear stress for combined wave-current motion is given by:

$$\tau_b = \frac{1}{2}\rho f_w \left(U_b^2 + U_\delta^2 + 2U_b U_\delta \cos\alpha\right) \quad (6.11)$$

Where

U_δ	Current velocity at top ($z = \delta_w$) of wave boundary layer
α	Angle between mean current and direction of wave propagation

The resulting bed shear stress is found by the largest value of the bed shear stress for pure current derived by Equation (6.1) and the value derived by Equation (6.11).

6.2 Viscosity and Density

These two processes impact the HD-module by impacting the density and viscosity.

6.2.1 Mud impact on density

The influence from the mud on the water density is by definition given by:

$$\rho_m = \rho_w + \sum_i \left(1 - \frac{\rho_w}{\rho_s}\right) c^i \quad (6.12)$$

6.2.2 Mud influence on viscosity

The influence on the kinematic viscosity from the mud can be parameterised by:

$$\frac{\upsilon_M}{\nu} = k_{v1}^{(a/k_{v2})} \quad (6.13)$$

Where k_{v1} and k_{v2} are calibration parameters and $a = \sum_i c^i$.

This expression is assumed to be valid for applying a lower limit for the eddy viscosity, hence:

$$\upsilon_T = \max\left(\upsilon_T, \upsilon_M\right) \tag{6.14}$$

Utilising

$$\frac{\upsilon_M}{\nu_T} = k_{v1}^{(a/k_{v2})} \tag{6.15}$$

7 Multi-Layer and Multi-Fraction Applications

7.1 Introduction

The MT model is a multi-layer and multi-fraction model. In the water column the mass concentrations c_1, c_2, etc. to c_i are defined. In the bed $c_{1,1}$, $c_{2,1}$, etc. to $c_{i,1}$ are defined for the first layer and $c_{1,2}$, $c_{2,2}$, etc. to $c_{i,2}$ for the second layer and consecutively. See also Figure 7.1.

Water column	Mass concentrations c_1 , c_2 ,..... c_i
Bed layer 1	Dry density $c_{1,1}$, $c_{2,1}$,..... $c_{i,1}$
Bed layer 2	Dry density $c_{1,2}$, $c_{2,2}$,..... $c_{i,2}$
Bed layer n	Dry density $c_{1,n}$, $c_{2,n}$,..... $c_{i,n}$

Figure 7.1 Definition sketch for multi fractions-layers

The fractions are defined by their sediment characteristics. For the cohesive sediment fractions this gives the following extensions to the formulae above for deposition and erosion.

The deposition for the *i* mud fraction is:

$$D^i = w_s^i \; c_b^i \; p_D^i \tag{7.1}$$

where p_D^i is a probability ramp function of deposition:

$$p_D^i = \max\left(0, \min\left(1, \quad 1 - \frac{\tau_b}{\tau_{cd}^i}\right)\right) \tag{7.2}$$

Erosion of the top layer of the bed is considered one incident calculated for one time step updating the sediment fraction ratio of the bed.

In the Mud Transport module, the sand transport description is based on the assumption that erosion takes place simultaneously for both sand and cohesive sediment. Therefore, the erosion of each layer is calculated using the normal mud transport erosion equations. Afterwards, the fraction of the sediment that may be kept in suspension under the present hydrodynamic conditions is calculated.

7.2 Dense Consolidated Bed

For a dense consolidated bed the erosion rate from the top layer *j*, can be calculated in the following way:

$$E_{j,total} = E_0^j \; p_E^{j^{E_m}} \tag{7.3}$$

where p_E^j is a probability ramp function of erosion and E_0 is the erodibility.

$$p_E^j = \max\left(0, \frac{\tau_b}{\tau_{ce}^j} - 1\right) \tag{7.4}$$

The erosion rate for the fraction *i* is then calculated as:

$$E_{i,j} = \frac{M_{i,j}}{M_{total,j}} E_{total}^j \tag{7.5}$$

in which *M* is the mass of sediment in the layer *j*.

7.3 Soft, Partly Consolidated Bed

Similarly for a soft, partly consolidated bed:

$$E_{total}^j = E_0^j \exp\left(\alpha^j \left(\tau_b - \tau_{ce}^j\right)^{0.5}\right) \tag{7.6}$$

The erosion rate for the fraction *i* is then calculated as:

$$E_{i,j} = \frac{M_{i,j}}{M_{total,j}} E_{total}^j \tag{7.7}$$

Each layer in the bed contains a certain concentration of sediment defined by a dry density excluding water content. This density is assigned the set of fractions applied. For example 60 percent particles < 63 μm and 40 percent fine sand. This ratio is not fixed but can vary throughout the simulations, dependent on the advection-dispersion processes. An account is kept of the sediment ratios for the bed. This allows for a grain sorting process to take place.

8 Morphological Features

8.1 Morphological Simulations

The morphological evolution is sought to be included by updating the bathymetry for every time step with the net sedimentation rate. This ensures a stable evolution in the model that will not destabilise the hydrodynamic simulation.

$$bat^{n+1} = bat^{n} + nested^{n} \tag{8.1}$$

where

bat^n	Bathymetry at present time step
bat^{n+1}	Bathymetry at next time step
n	Time step

The Mud Transport module also allows the morphological evolution to be speeded up in the following way.

$$bat^{n+1} = bat^{n} + nested^{n}\ Speedup \tag{8.2}$$

Speedup is a dimensionless factor. This factor is relevant for cases where the sedimentation processes are governed by cyclic events, such as tides or seasonal variation. This does NOT apply to stochastic events, such as storms.

The thickness of the individual bed layers is updated at the same time as the bathymetry. This is not the case for the suspended concentration.

8.2 Bed Update

The bed layer is updated using the following logistics (only 1 layer and 1 fraction is considered)

1. The net deposition is calculated as $ND = \sum_{i=0}^{I} \left(D^i - E^i\right)\Delta t$
2. The bed mass M is calculated.
3. If net erosion occurs ($ND > 0$) and $M + ND < 0$, the deposition and erosion rates are adjusted such that $M + ND = 0$.

4. The bed layer thickness, H, and density, ρ, are updated as:

$$H_{bed}^{new} = \begin{cases} H_{bed}^{old} + \dfrac{ND \cdot \Delta t}{\rho^{i}} & , \quad ND \le 0 \\ \\ H_{bed}^{old} + \dfrac{ND \cdot \Delta t}{\rho_{bed}^{old}} & , \quad ND > 0 \end{cases} \tag{8.3}$$

$$\rho_{bed}^{new} = \frac{H_{bed}^{old} \cdot \rho_{bed}^{old} + ND \cdot \Delta t}{H_{bed}^{new}} \tag{8.4}$$

9 References

/1/ Andersen, T.J., 2001.
Seasonal variation in erodibility of two temperate microtidal mudflats. Estuarine, Coastal and Shelf Science 56, pp 35-46

/2/ Andersen, T.J. & Pejrup, M., 2001.
Suspended sediment transport on a temperate, microtidal mudflat, the Danish Wadden Sea. Marine Geology 173, pp 69-85

/3/ Burt, T.N., 1986.
Field settling velocities of estuary muds. In Estuarine Cohesive Sediment Dynamics. Lecture Notes on Coastal and Estuarine Studies. (Mehta, A.J., ed.). Springer Verlag, Berlin, pp 126-150.

/4/ Delo, E.A., Ockenden, M.C., 1992.
"Estuarine Muds Manual" HR Wallingford, Report SR309, May 1992.

/5/ Dyer, K.R., 1986.
Coastal and Estuarine Sediment Dynamics. John Wiles & Sons, pp 342.

/6/ Edelvang, K. & Austen, I., 1997.
The temporal variation of flocs and fecal pellets in a tidal channel. Estuarine, Coastal and Shelf Science 44, 361-367.

/7/ Eisma, D., 1993.
Suspended matter in the aquatic environment. Springer Verlag, pp 315.

/8/ Ekebjærg, L. and Justesen, P., 1991.
"An Explicit Scheme for Advection-Diffusion Modelling in Two Dimensions". Comp. Meth. App. Mech. Eng., pp. 287-297.

/9/ Engelund, F., Fredsøe, J., 1976.
"A sediment transport model for straight alluvial channels". Nordic Hydrology 7, pp. 293 - 306.

/10/ Fenton and McKee, 1990.
"On Calculating the Lengths of Water Waves". Coastal Engineering, Vol. 14, Elsevier Science Publishers BV, Amsterdam, pp. 499-513.

/11/ Fredsøe, J., 1981.
"Mean Current Velocity Distribution in Combined Waves and Current". Progress Report No. 53, ISVA, Technical University of Denmark.

/12/ Fredsøe, J., 1984.
"Turbulent boundary layer in wave-current motion". J Hydr Eng, A S C E, Vol 110, HY8, 1103-1120.

/13/ Gibbs, R.J. 1985.
Estuarine flocs: Their size, settling velocity and density. Journal of Geophysical Research 90, pp 3249-3251.

/14/ Grishanin, K.V, and Lavygin, A.M., 1987.
"Sedimentation of dredging cuts in sand bottom rivers". PIANC, Bulletin, No. 59, pp. 50-55.

/15/ Jonsson, I.G. and Carlsen, N.A., 1976.
"Experimental and Theoretical Investigations in an Oscillatory Rough Turbulent Boundary Layer". J. Hydr. Res. Vol. 14, No. 1, pp. 45-60.

/16/ Krone, R.B., 1962.
"Flume Studies of the Transport of Sediment in Estuarial Processes". Hydraulic Engineering Laboratory and Sanitary Engineering Research Laboratory, Univ. of California, Berkely, California, Final Report.

/17/ Krone, B. 1986.
The significance of aggregate properties to transport processes. In: Mehta A.J. (Ed) Estuarine Cohesive Sediment Dynamics, Springer Verlag, pp. 66-84.

/18/ Leonard, B.P., 1979.
"A stable and accurate convective modelling procedure based on quadratic upstream interpolation". Comput. Meths. Appl. Mech. Eng. 19, pp. 59-98.

/19/ Leonard, B.P., 1991.
"The ULTIMATE Conservative Differential Scheme Applied to Unsteady One-Dimensional Advection". Comput. Meths. Appl. Mech. Eng. 88, 17-74.

/20/ Mehta, A.J., Hayter, E.J., Parker, W.R., Krone, R.B., Teeter, A.M., 1989.
"Cohesive Sediment Transport. I: Process Description". Journal of Hydraulic Eng., Vol. 115, No. 8, Aug. 1989, pp. 1076-1093.

/21/ Parchure, T. M. and Mehta, A.J., 1985.
Erosion of soft cohesive sediment deposits. Journal of Hydraulic Engineering - ASCE 111 (10), 1308-1326.

/22/ Partheniades, E., 1965.
Erosion and deposition of cohesive soils. Journal of the Hydraulics Division Proceedings of the ASCE 91 (HY1), 105-139.

/23/ Pejrup, M., 1988a.
Flocculated suspended sediment in a micro-tidal environment. Sedimentary Geology 57, 249-256.

/24/ Pejrup, M., Larsen, M. and Edelvang, K., 1997.
A fine-grained sediment budget for the Sylt-Rømø tidal basin. Helgoländer Meeresuntersuchungen 51, 1-15.

/25/ Postma, H., 1967.
"Sediment Transport and Sedimentation in the Estuarine Environment". In: Lauff G.H: (Ed) Estuaries AAAS Publ. 83, p 158-179

/26/ Rijn, L.C., 1984.
"Sediment Transport, Part I Bed Load Transport". Journal of Hydraulic Engineering, Vol. 110, No. 10, October, 1984.

/27/ Rijn, L.C., 1984.
"Sediment Transport, Part II Suspended Load Transport". Journal of Hydraulic Engineering, Vol. 110, No. 10, October, 1984.

/28/ Rijn, Van L.C., 1989.
"Handbook on Sediment Transport by Current and Waves". Delft Hydraulics, Report H461, June 1989, pp. 12.1-12.27.

/29/ Sanford, L.P. and Maa, J.P., 2001.
A unified erosion formulation for fine sediments. Marine Geology 179 (1-2), 9-23.

/30/ Soulsby R.L., Hamm L., Klopman G., Myrhaug D., Simons R.R., and Thomas G.P., 1993.
"Wave-current interaction within and outside the bottom boundary layer". Coastal Engineering, 21, 41-69.

/31/ Swart, D.H., 1974.
"Offshore sediment transport and equilibrium beach profiles". Delft Hydr. Lab. Publ., 1312, Delft Univ. technology Diss., Delft.

/32/ Teeter, A.M., 1986.
"Vertical Transport in Fine-Grained Suspension and Nearly-Deposited Sediment". Estuarine Cohesive Sediment Dynamics, Lecture Notes on Coastal and Estuarine Studies, 14, Springer Verlag, pp. 126-149.

/33/ Teisson, C., 1991.
"Cohesive suspended sediment transport: feasibility and limitations of numerical modelling". Journal of Hydraulic Research, Vol. 29, No. 6.

/34/ Van Leussen, W. 1988.
Aggregation of particles, settling velocity of mud flocs. A review: In: Dronkers & Van Leussen (Eds.): Physical processes in estuaries. Springer Verlag, pp 347-403.

/35/ Van Olphen, H. 1963.
"An introduction to clay colloid chemistry. ". Interscience Publishers (John Wiley & Sons) pp 301.

/36/ Van Straaten, L.M.J.U. and Kuenen, Ph.H., 1958.
Tidal action as a cause of clay accumulation. Journal of Sedimentary Petrology 28 (4), 406-413.

/37/ Whitehouse, U.G., Jeffrey, L.M., Debbrecht, J.D. 1960.
Differential settling tendencies of clay minerals in saline waters. In: Swineford (Ed) Clays and clay minerals. Proceedings 7th National Conference, Pergamon Press, p 1-79.

/38/ Yalin, M.S, 1972.
"Mechanics of Sediment Transport". Pergamon Press Ltd. Hendington Hill Hall, Oxford.